Student name: ______________________

Class/Teacher name: ________________

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Sydney Technical High School

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time - 180 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- In questions 11-40, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 1-5)

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7-31)

- Attempt questions 11-40
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

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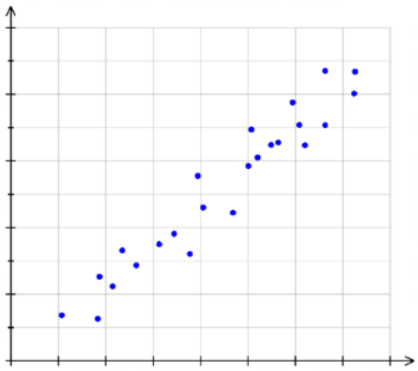
1. What is the indefinite integral of $\frac{1}{x^3}$?

(A) $-\frac{3}{x^4}+c$

(B) $-\frac{1}{3x^4}+c$

(C) $-\frac{1}{2x^2}+c$

(D) $-\frac{1}{3x^2}+c$

2. 

What is the correlation between the variables in this scatterplot?

(A) Weak positive

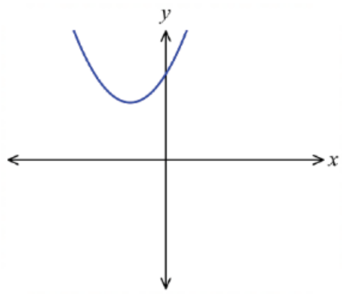
(B) Weak negative

(C) Moderate positive

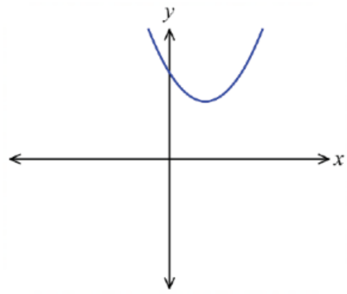
(D) Moderate negative

3. Which diagram could be the graph of the parabola $y = 2 - (x+1)^2$?

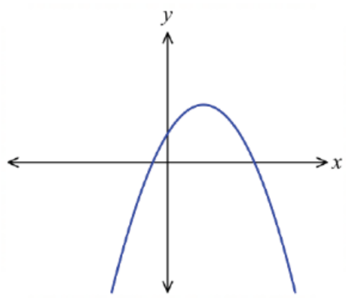
(A)



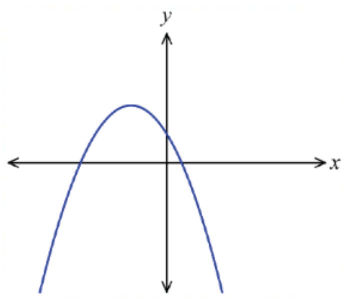
(B)



(C)



(D)



4. What is the domain of the function $y = \frac{1}{\sqrt{x-9}}$?

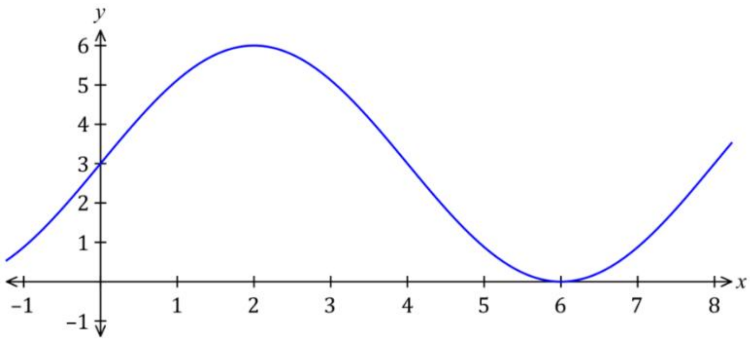
(A) Domain: [9, ∞)

(B) Domain: (9, ∞)

(C) Domain: $(-\infty, \infty)$

(D) Domain: $[-3, 3]$

5.



Which of the following equations is likely to be the rule for the graph of the trigonometric function shown above?

(A) $y = 3 + 3\sin\left(\frac{\pi x}{4}\right)$

(B) $y = 3 + 3\cos\left(\frac{\pi x}{4}\right)$

(C) $y = 3 + 3\sin\left(\frac{x}{4}\right)$

(D) $y = 3 + 3\cos\left(\frac{x}{4}\right)$

6. The probability density function for a continuous random variable X is:

$$f(x) = \begin{cases} sinx & 0 < x < k \\ 0 & \text{otherwise} \end{cases}$$

What is the value of k?

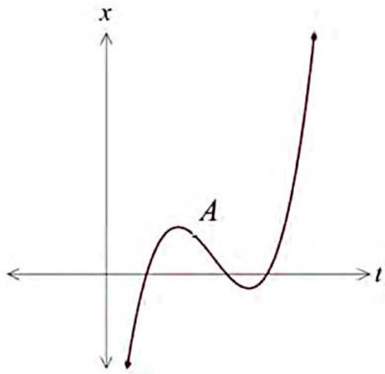
(A) $\frac{\pi}{2}$

(B) π

(C) 1

(D) 2

7. The diagram shows the displacement, x metres, of a moving object at time t seconds.



Which of the following statements describes the motion of the object at the point A

(A) Velocity is negative and acceleration is positive

(B) Velocity is negative and acceleration is negative

(C) Velocity is positive and acceleration is negative

(D) Velocity is positive and acceleration is positive

8. The derivative of $e^{-4x}cos2x$ with respect to x is:

(A) $e^{-4x}(\sin 2x - 2\cos 2x)$

(B) $2e^{-4x}(\sin 2x + 2\cos 2x)$

(C) $-e^{-4x}(\sin 2x - 2\cos 2x)$

(D) $-2e^{-4x}(\sin 2x + 2\cos 2x)$

9. Let $a = e^x$. Which expression is equal to $log_e(a^2)$?

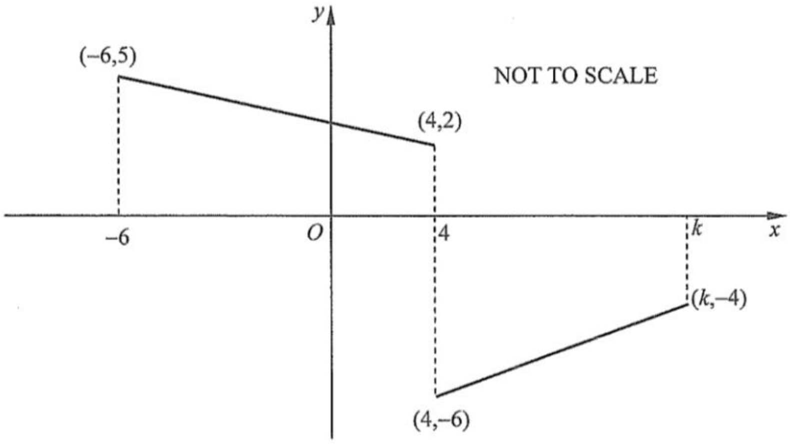
(A) e^{2x}

(B) e^{x^2}

(C) $2x$

(D) x^2

10.



Use the graph above to find the value of *k* which satisfies $\int_{-6}^{k} f(x)dx = 0$

(A) 6

(B) 10

(C) 11

(D) 12

Mathematics Advanced

Section II Answer Booklet

90 marks

Attempt Questions 11-40

Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.

Please turn over

Question 11 (2 marks)

Simplify $\frac{x}{x^2-4} - \frac{4}{x-2}$ 2

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Question 12 (1 mark)

Rationalise the denominator of $\frac{7}{\sqrt{5}-2}$ 1

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Question 13 (2 marks)

Find the equation of the line that passes through the point $(0, -3)$ and has an angle of inclination of $30°$. Leave your answer in gradient-intercept form. 2

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Question 14 (2 marks)

Differentiate the following with respect to x

(a) $f(x) = tan7x$ 1

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(b) $f(x) = ln(x^2 + 2)$ 1

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Question 15 (1 mark)

Find $\int (4x + 3)^9 dx$ 1

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Question 16 (3 marks)

The number of students absent from Year 12 for the past nine days was as follows:
14, 17, 13, 16, 17, 12, 11, 28, 19

(a) Find the standard deviation. Answer correct to one decimal place. 1

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(b) Find the lower quartile and the upper quartile 1

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(c) Find the interquartile range 1

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Question 17 (2 marks)

Solve the equation $tan^2x = 3$ for $0 \leq x \leq 2\pi$ 2

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Question 18 (2 marks)

There are fifteen marbles in a jar. Five of the marbles are red, five are blue and five are yellow. Ron randomly selects two marbles and puts them in his pocket.

(a) What is the probability that the two marbles are red? 1

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(b) What is the probability that the two marbles are the same colour? 1

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Question 19 (4 marks)

For the following continuous probability distribution

$$f(x) = \begin{cases} \frac{3x^2}{511} \text{ for } 1 \leq x \leq 8 \\ 0 \text{ otherwise} \end{cases}$$

(a) Find $P(X = 5)$ 1

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Question 19 continues on page 13

Question 19 continued

(b) Find $P(X \leq 5)$ 2

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(c) Find $P(X > 5)$ 1

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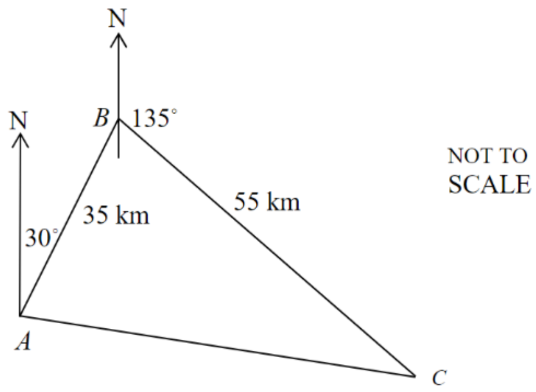
Question 20 (3 marks)

A motorist drives 35 km from Town *A* to Town *B* on a bearing of $030°T$.

He then drives 55 km to Town *C* that is on a bearing of $135°T$ from Town *B.*



(a) Find the size of $\angle ABC$ 1

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(b) Find the distance between *A* and *C* to the nearest kilometre. 2

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Question 21 (2 marks)

In a class of 23 students, on Sunday night 12 watched The Office, 13 watched Stranger Things while 7 watched both.

(a) Find the probability that a student chosen at random watched neither 1

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Question 21 continues on page 15

Question 21 continued

(b) Find the probability that they watched The Office, given that they watched Stranger Things 1

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Question 22 (2 marks)

Find the exact value of $\int_0^{\frac{\pi}{3}} \left(1 - sec^2 \frac{x}{2}\right) dx$ 2

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Question 23 (1 mark)

Consider the function $f(x) = 4 + 3cos\left(\frac{\pi x}{2}\right)$

State the amplitude 1

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Question 24 (2 marks)

Simplify $\frac{log(a^3b^2) - log(ab^2)}{log\sqrt{a}}$ 2

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Question 25 (3 marks)

Prove that $\frac{cos\theta}{1+sin\theta} + tan\theta = sec\theta$ 3

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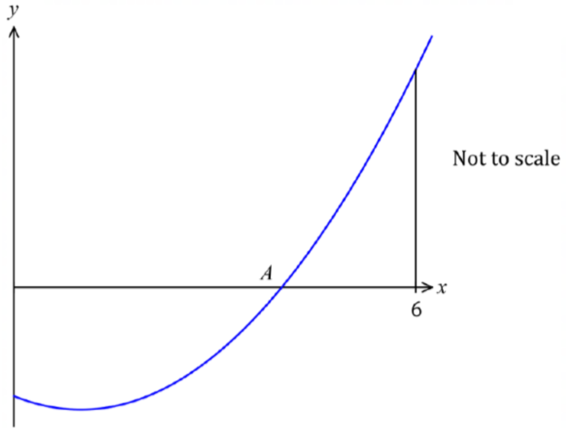
Question 26 (4 marks)

The diagram below shows the graph of $y = x^2 - 2x - 8$



(a) What are the coordinates of A? 1

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(b) Find the area bounded by the x-axis and the curve $y = x^2 - 2x - 8$ between $0 \leq x \leq 6$ 3

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Question 27 (6 marks)

Consider the function $f(x) = x^3 - 3x^2 - 9x + 6$

(a) Show that stationary points occur at $(-1, 11)$ and $(3, -21)$ and determine their nature 3

Question 27 continues on page 19

Question 27 continued

(b) Find the coordinates of any points of inflexion **2**

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(c) Sketch the graph of $y = f(x)$, labelling the stationary points and the y-intercept. Do not attempt to find the x-intercepts. **1**

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Question 28 (2 marks)

Differentiate $log_3 x^3$ 2

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Question 29 (1 mark)

$\int \frac{x}{x^2-5}\ dx$ 1

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Question 30 (3 marks)

Determine the equation of a curve given by $\frac{d^2y}{dx^2} = 18x + 4$ if it is known that $(1, -2)$ is a stationary point on the curve. 3

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Question 31 (3 marks)

(a) Sketch $y = 3cosx$ in the domain $-2\pi \leq x \leq 2\pi$ 2

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(b) Hence or otherwise, how many solutions are there to the equation $3cosx = 1 - x$? 1

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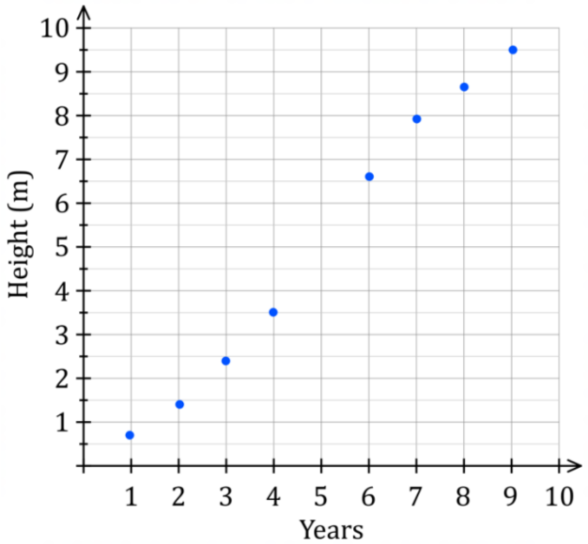
Question 32 (4 marks)

Tim is a scientist studying the growth of a particular tree over several years. The data he recorded is shown in the table below.

Years since planting, t	1	2	3	4	6	7	8	9
Height of tree, H metres	0.7	1.4	2.4	3.5	6.6	7.9	8.7	9.5

A scatterplot of the data is shown below.



(a) What is Pearson's correlation coefficient? Answer correct to 4 decimal places. 1

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(b) Find the equation of the least-squares line of best fit in terms of years (t) and height (h). Answer correct to 2 decimal places. 1

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(c) Tim did not record the tree's height after five years. Predict the height after five years, correct to 1 decimal place. 1

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(d) Estimate how many years it will take for the tree to reach a height of 24 metres. Answer correct to 1 decimal place. 1

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Question 33 (2 marks)

An object is in motion along a line. The velocity, as measured at several instants of time, is given in the following table. Use the trapezoidal rule with 5 function values to approximate the distance travelled from $t = 0$ to $t = 4$ seconds. 2

t (s)	0	1	2	3	4
v (m/s)	3.2	2.7	2.9	4.0	4.7

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Question 34 (3 marks)

A large tank of liquid which contains L litres of a toxic chemical is being drained.
The amount of chemical in the tank over time t minutes, is given by:

$$L = 110(20 - t)^2$$

(a) At what rate is the chemical draining out of the tank after 5 minutes? 2

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(b) How long will it take for the tank to be completely empty? 1

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Question 35 (4 marks)

A factory is producing laptops. The annual production, M laptops at time t years, is given by:

$M = M_0 e^{kt}$

Initially the production at the factory was 2000 laptops per annum.

Five years later it had increased to 3200 laptops per annum.

(a) Find the values of M_0 and k (Answer correct to three decimal places). 2

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(b) How many years will it take for the production to double its original output? Answer correct to one decimal place. 2

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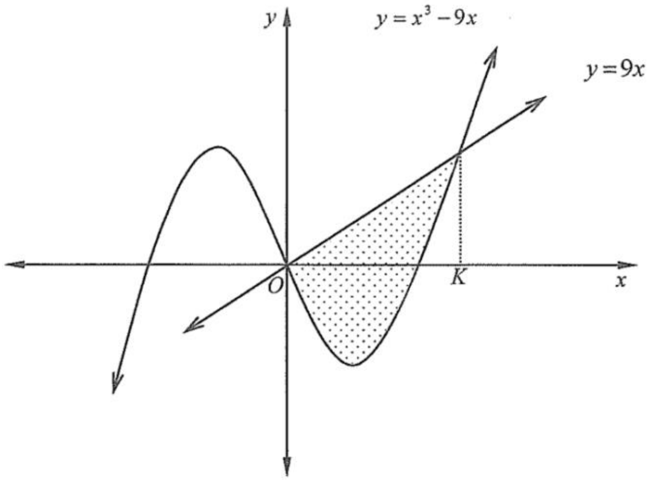
Question 36 (5 marks)



The graphs of the functions $y = 9x$ and $y = x^3 - 9x$ are shown above.

The graphs intersect at $x = 0$ and $x = K$ for $x \geq 0$.

(a) By solving the two equations simultaneously, show that $K = 3\sqrt{2}$ 2

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(b) Hence find the shaded area in the diagram above. 3

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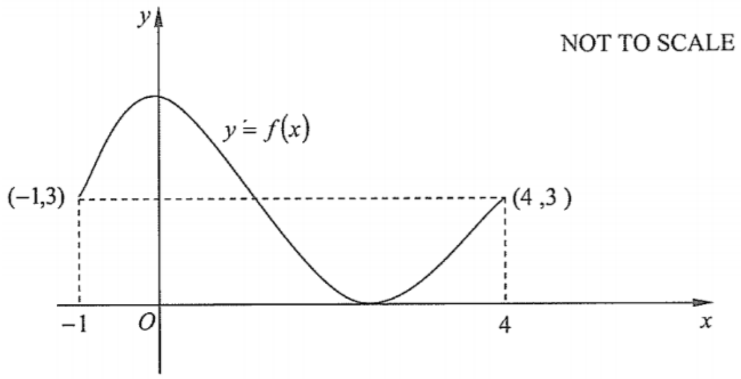
Question 37 (2 marks)

The graph below represents the function $y = f(x)$.



If $\int_{-1}^{4} f(x)\,dx = \frac{15}{2}$, find the value of $\int_{-1}^{4} [f(x) + 4]\,dx$ 2

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Question 38 (4 marks)

(a) Show that $\frac{1}{2x-5} - \frac{1}{2x+5} = \frac{10}{4x^2-25}$ 1

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(b) Hence find $\int \frac{dx}{4x^2-25}$ Leave your answer in simplest form. 3

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Question 39 (8 marks)

A particle is moving in a straight line and its velocity is given by:

$$v = 1 - 2sin2t \text{ for } 0 \leq t \leq \pi$$

where v is measured in metres per second and t in seconds.

(a) Sketch the graph of v as a function of t for $0 \leq t \leq \pi$ 2

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(b) At what time(s) is particle's acceleration zero? 1

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(c) When is the particle at rest? 2

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Question 39 continues on page 29

Question 39 continued

(d) Initially the particle is at the origin.
Find the displacement function x as a function of t 2

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(e) What is the position of the particle when $t = \frac{\pi}{3}$? Leave your answer in exact form. 1

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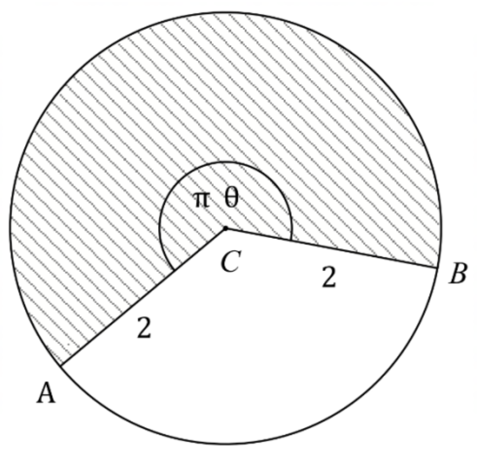
Question 40 (7 marks)



The angle at the centre *C* of a circle of radius 2 cm is $\pi\theta$ radians, $0 < \theta < 2$, as shown on the diagram above.

(a) Write down the length of the arc of the shaded sector 1

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(b) The sector is cut from the circle along the radii *CA* and *CB* and folded to make a cone. 1
Find the radius of the cone.

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(c) Show that the volume of the cone is given by $V = \frac{\pi}{3}\sqrt{4\theta^4 - \theta^6}$ 1

(The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2\sqrt{l^2 - r^2}$ where l is the slant height)

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Question 40 continues on page 31

Question 40 continued

(d) Find the value of θ, to 2 decimal places, for which the volume of the cone is maximised. 4

End of paper

Student name: ________________

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Sydney Technical High School

Mathematics Advanced

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Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

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(A) $-\frac{3}{x^4}+c$

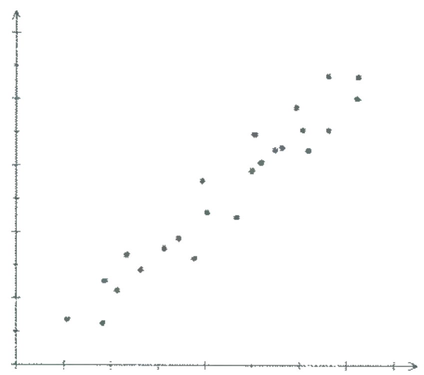
(B) $-\frac{1}{3x^4}+c$

(C) $-\frac{1}{2x^2}+c$

(D) $-\frac{1}{3x^2}+c$

$$\int x^{-3}\,dx = \frac{x^{-2}}{-2}+C = -\frac{1}{2x^2}+C$$

2.



What is the correlation between the variables in this scatterplot?

(A) Weak positive

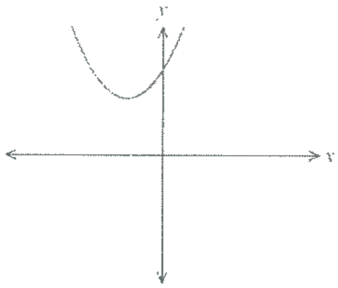
(B) Weak negative

(C) Moderate positive

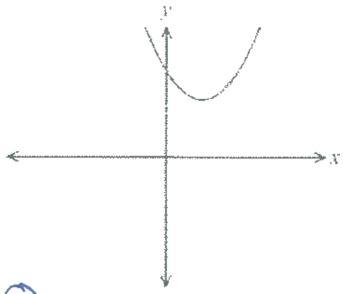
(D) Moderate negative

3. Which diagram could be the graph of the parabola $y = 2 - (x+1)^2$?

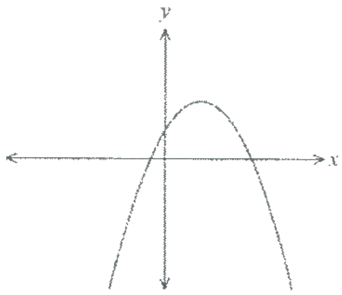
(A)



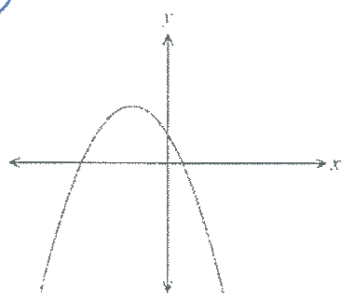
(B)



(C)



(D)



4. What is the domain of the function $y = \frac{1}{\sqrt{x-9}}$?

(A) Domain: $[9, \infty)$

(B) Domain: $(9, \infty)$

(C) Domain: $(-\infty, \infty)$

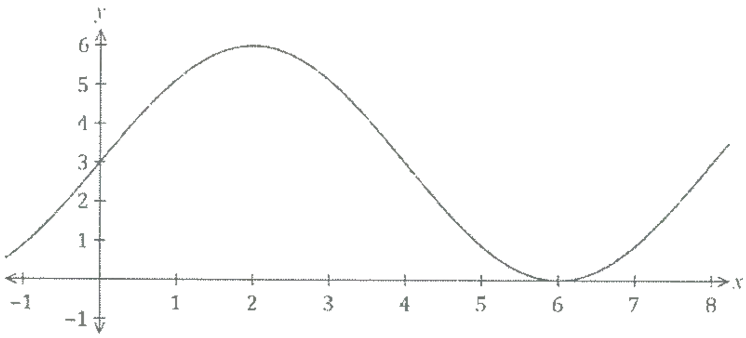
(D) Domain: $[-3, 3]$

$x - 9 > 0$

$x > 9$

$(9, \infty)$

5.



Which of the following equations is likely to be the rule for the graph of the trigonometric function shown above?

(A) $y = 3 + 3\sin\left(\frac{\pi x}{4}\right)$

(B) $y = 3 + 3\cos\left(\frac{\pi x}{4}\right)$

(C) $y = 3 + 3\sin\left(\frac{x}{4}\right)$

(D) $y = 3 + 3\cos\left(\frac{x}{4}\right)$

Period $= 8$

$\frac{2\pi}{b} = 8$

$2\pi = 8b$

$b = \frac{\pi}{4}$

6. The probability density function for a continuous random variable X is:

$$f(x) = \begin{cases} \sin x & 0 < x < k \\ 0 & \text{otherwise} \end{cases}$$

What is the value of k?

(A) $\frac{\pi}{2}$

(B) π

(C) 1

(D) 2

$\int_0^k \sin x = 1$

$[-\cos x]_0^k = 1$

$-[\cos k - \cos 0] = 1$

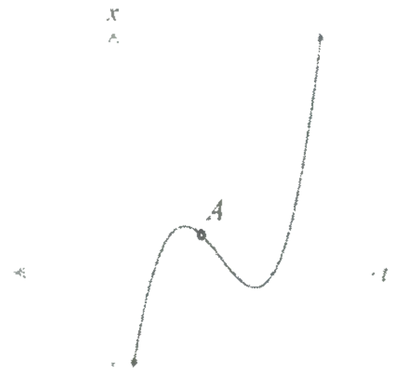
$-[\cos k - 1] = 1$

$\cos k - 1 = -1$

$\cos k = 0$

$k = \frac{\pi}{2}$

7. The diagram shows the displacement, x metres, of a moving object at time t seconds.



Which of the following statements describes the motion of the object at the point A

(A) Velocity is negative and acceleration is positive

(B) Velocity is negative and acceleration is negative

(C) Velocity is positive and acceleration is negative

(D) Velocity is positive and acceleration is positive

8. The derivative of $e^{-4x}cos2x$ with respect to x is:

(A) $e^{-4x}(\sin 2x - 2\cos 2x)$

(B) $2e^{-4x}(\sin 2x + 2\cos 2x)$

(C) $-e^{-4x}(\sin 2x - 2\cos 2x)$

(D) $-2e^{-4x}(\sin 2x + 2\cos 2x)$

$\frac{d}{dx}\left(e^{-4x}\cos 2x\right) = -4e^{-4x}\cos 2x - 2e^{-4x}\sin 2x$

$= -2e^{-4x}[2\cos 2x + \sin 2x]$

9. Let $a = e^x$. Which expression is equal to $log_e(a^2)$?

(A) e^{2x}

(B) e^{x^2}

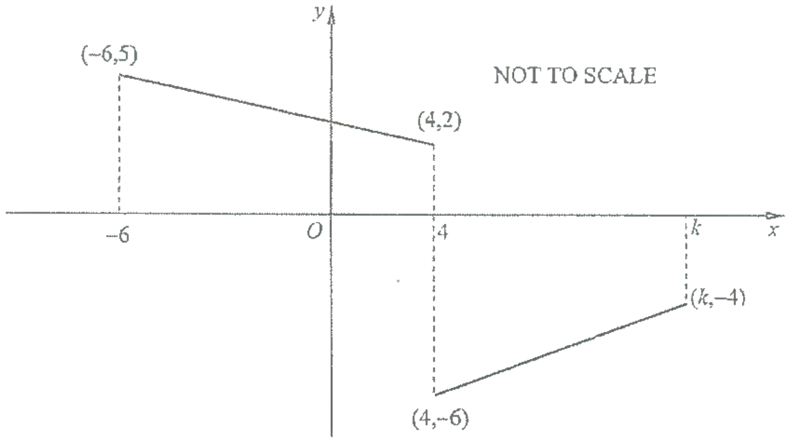
(C) $2x$

(D) x^2

$\log_e(e^x)^2 = \log_e e^{2x}$

$= 2x$

10.



Use the graph above to find the value of k which satisfies $\int_{-6}^{k} f(x)dx = 0$

(A) 6

(B) 10

(C) 11

(D) 12

Area trapezium above $= \frac{1}{2} \times 10(2+5)$

$= 35$

Area trapezium below: $35 = \frac{1}{2}(k-4)(4+6)$

$35 = 5(k-4)$

$7 = k-4$

$k = 11$

Question 11 (2 marks)

Simplify $\frac{x}{x^2-4} - \frac{4}{x-2}$ 2

$= \frac{x}{(x-2)(x+2)} - \frac{4}{x-2}$

$= \frac{x-4(x+2)}{(x-2)(x+2)}$

$= \frac{x-4x-8}{(x-2)(x+2)}$

$= \frac{-3x-8}{(x-2)(x+2)}$

Question 12 (1 mark)

Rationalise the denominator of $\frac{7}{\sqrt{5}-2}$ 1

$= \frac{7}{(\sqrt{5}-2)} \times \frac{(\sqrt{5}+2)}{(\sqrt{5}+2)}$

$= \frac{7(\sqrt{5}+2)}{5-4}$

$= 7(\sqrt{5}+2)$

Question 13 (2 marks)

Find the equation of the line that passes through the point $(0, -3)$ and has an angle of inclination of $30°$. Leave your answer in gradient-intercept form. 2

$m = \tan\theta$ $(0, -3)$

$= \tan 30°$

$= \frac{1}{\sqrt{3}}$

$y = \frac{1}{\sqrt{3}}x - 3$

Question 14 (2 marks)

Differentiate the following with respect to x

(a) $f(x) = tan7x$ 1

$f'(x) = 7\sec^2 7x$

(b) $f(x) = ln(x^2 + 2)$ 1

$f'(x) = \frac{2x}{x^2+2}$

Question 15 (1 mark)

Find $\int (4x + 3)^9 dx$ 1

$= \frac{(4x+3)^{10}}{10 \times 4} + c$

$= \frac{(4x+3)^{10}}{40} + c$

Question 16 (3 marks)

The number of students absent from Year 12 for the past nine days was as follows:
14, 17, 13, 16, 17, 12, 11, 28, 19

(a) Find the standard deviation. Answer correct to one decimal place. 1

$\sigma_x = 4.8$

(b) Find the lower quartile and the upper quartile 1

11 12 ↑ 13 14 (16) 17 17 ↑ 19 28

$Q_1 = 12.5$ Q_2 $Q_3 = 18$

∴ Lower quartile is 12.5
Upper quartile is 18

(c) Find the interquartile range 1

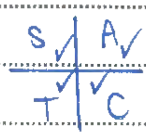
$IQR = 18 - 12.5$
$= 5.5$

Question 17 (2 marks)

Solve the equation $tan^2x = 3$ for $0 \leq x \leq 2\pi$ 2

$(\tan x)^2 = 3$

$\tan x = \pm\sqrt{3}$



related angle $= \frac{\pi}{3}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

Question 18 (2 marks)

There are fifteen marbles in a jar. Five of the marbles are red, five are blue and five are yellow. Ron randomly selects two marbles and puts them in his pocket.

(a) What is the probability that the two marbles are red? 1

$P(2 \text{ red}) = \frac{5}{15} \times \frac{4}{14}$

$= \frac{2}{21}$

(b) What is the probability that the two marbles are the same colour? 1

$P(2 \text{ same colour}) = P(RR) + P(BB) + P(YY)$

$= \frac{2}{21} + \frac{2}{21} + \frac{2}{21}$

$= \frac{2}{7}$

Question 19 (4 marks)

For the following continuous probability distribution

$$f(x) = \begin{cases} \frac{3x^2}{511} \text{ for } 1 \leq x \leq 8 \\ 0 \text{ otherwise} \end{cases}$$

(a) Find $P(X = 5)$ 1

$P(X = 5) = 0$

Question 19 continues on page 11

Question 19 continued

(b) Find $P(X \leq 5)$ 2

$$P(X \leq 5) = \int_1^5 \frac{3x^2}{511}\,dx$$

$$= \frac{1}{511}\int_1^5 3x^2\,dx$$

$$= \frac{1}{511}\left[x^3\right]_1^5$$

$$= \frac{1}{511}\left[5^3 - 1^3\right]$$

$$= \frac{1}{511}\left[124\right]$$

$$= \frac{124}{511}$$

(c) Find $P(X > 5)$ 1

$$P(X > 5) = 1 - P(X \leq 5)$$

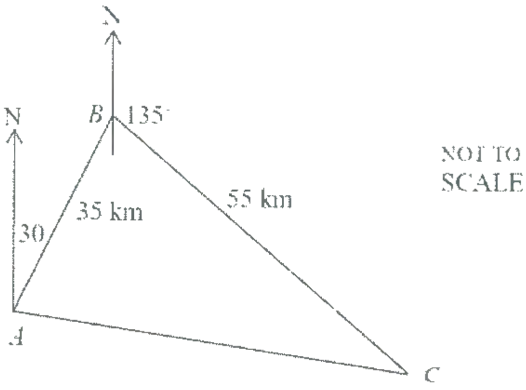
$$= 1 - \frac{124}{511}$$

$$= \frac{387}{511}$$

Question 20 (3 marks)

A motorist drives 35 km from Town A to Town B on a bearing of $030°T$.

He then drives 55 km to Town C that is on a bearing of $135°T$ from Town B.

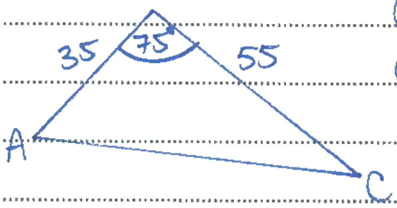


(a) Find the size of $\angle ABC$ 1

$\angle ABC = 30° + (180° - 135°)$

$= 75°$

(b) Find the distance between A and C to the nearest kilometre. 2



$(AC)^2 = 35^2 + 55^2 - 2 \times 35 \times 55 \cos 75°$

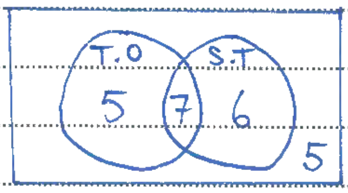
$(AC)^2 = 3253.54...$ $AC > 0$

$AC = 57 km$

Question 21 (2 marks)

In a class of 23 students, on Sunday night 12 watched The Office, 13 watched Stranger Things while 7 watched both.

(a) Find the probability that a student chosen at random watched neither 1



$P(\text{neither}) = \frac{5}{23}$

Question 21 continues on page 13

Question 21 continued

(b) Find the probability that they watched The Office, given that they watched Stranger Things 1

P(The Office \ Stranger Things) $= \frac{7}{13}$

Question 22 (2 marks)

Find the exact value of $\int_0^{\frac{\pi}{3}} \left(1 - sec^2 \frac{x}{2}\right) dx$ 2

$= \left[x - \frac{1}{(\frac{1}{2})} \tan\frac{x}{2}\right]_0^{\frac{\pi}{3}}$

$= \left[x - 2\tan\frac{x}{2}\right]_0^{\frac{\pi}{3}}$

$= \left[\frac{\pi}{3} - 2\tan\frac{\pi}{6}\right] - \left[0 - 2\tan 0\right]$

$= \frac{\pi}{3} - 2 \times \frac{1}{\sqrt{3}}$

$= \frac{\pi}{3} - \frac{2}{\sqrt{3}}$

Question 23 (1 mark)

Consider the function $f(x) = 4 + 3cos\left(\frac{\pi x}{2}\right)$

State the amplitude 1

Amplitude = 3

Question 24 (2 marks)

Simplify $\frac{log(a^3b^2) - log(ab^2)}{log\sqrt{a}}$ 2

$= \frac{\log\left(\frac{a^3b^2}{ab^2}\right)}{\log a^{\frac{1}{2}}}$

$= \frac{\log(a^2)}{\log a^{\frac{1}{2}}}$

$= \frac{2\log a}{\frac{1}{2}\log a}$

$= 4$

Question 25 (3 marks)

Prove that $\frac{cos\theta}{1+sin\theta} + tan\theta = sec\theta$ 3

$LHS = \frac{\cos\theta}{1+\sin\theta} + \tan\theta$

$= \frac{\cos\theta}{1+\sin\theta} + \frac{\sin\theta}{\cos\theta}$

$= \frac{\cos^2\theta + \sin\theta(1+\sin\theta)}{(1+\sin\theta)\cos\theta}$

$= \frac{\cos^2\theta + \sin\theta + \sin^2\theta}{(1+\sin\theta)\cos\theta}$

$= \frac{1+\sin\theta}{(1+\sin\theta)\cos\theta}$ as $\cos^2\theta + \sin^2\theta = 1$

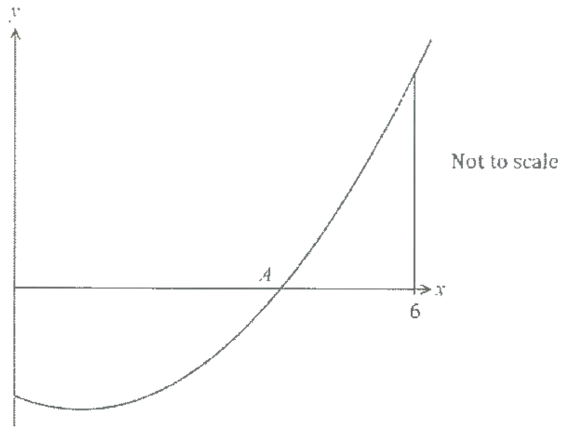
$= \frac{1}{\cos\theta}$

$= \sec\theta$

$= RHS$

Question 26 (4 marks)

The diagram below shows the graph of $y = x^2 - 2x - 8$



(a) What are the coordinates of A? 1

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4, -2$

$\therefore A(4,0)$

(b) Find the area bounded by the x-axis and the curve $y = x^2 - 2x - 8$ between $0 \le x \le 6$ 3

$A = \left|\int_0^4 x^2 - 2x - 8\, dx\right| + \int_4^6 x^2 - 2x - 8\, dx$

$= \left|\left[\frac{x^3}{3} - x^2 - 8x\right]_0^4\right| + \left[\frac{x^3}{3} - x^2 - 8x\right]_4^6$

$= \left|\left[\frac{4^3}{3} - 4^2 - 8(4)\right]\right| + \left[\left(\frac{6^3}{3} - 6^2 - 8(6)\right) - \left(\frac{4^3}{3} - 4^2 - 8(4)\right)\right]$

$= \left|-\frac{80}{3}\right| + \left[-12 - \left(-\frac{80}{3}\right)\right]$

$= \frac{124}{3}$

$= 41\frac{1}{3}$ units^2

Question 27 (6 marks)

Consider the function $f(x) = x^3 - 3x^2 - 9x + 6$

(a) Show that stationary points occur at $(-1, 11)$ and $(3, -21)$ and determine their nature 3

Stationary points occur when $f'(x)=0$

$f'(x) = 3x^2 - 6x - 9$

i.e. $3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$3(x-3)(x+1) = 0$

$x = 3, x = -1$

when $x = 3, y = -21$

when $x = -1, y = 11$

Check nature: $f''(x) = 6x - 6$

at $x = 3$, $f''(3) = 6(3) - 6$

$= 12 > 0$

$\therefore$ Minimum turning point at $(3, -21)$

at $x = -1$, $f''(-1) = 6(-1) - 6$

$= -12 < 0$

$\therefore$ Maximum turning point at $(-1, 11)$

(b) Find the coordinates of any points of inflexion 2

Points of inflexion occur when $f''(x) = 0$

i.e. $6x - 6 = 0$

$x = 1$

when $x = 1, y = -5$

$\therefore$ Possible point of inflexion at $(1, -5)$

Check concavity changes:

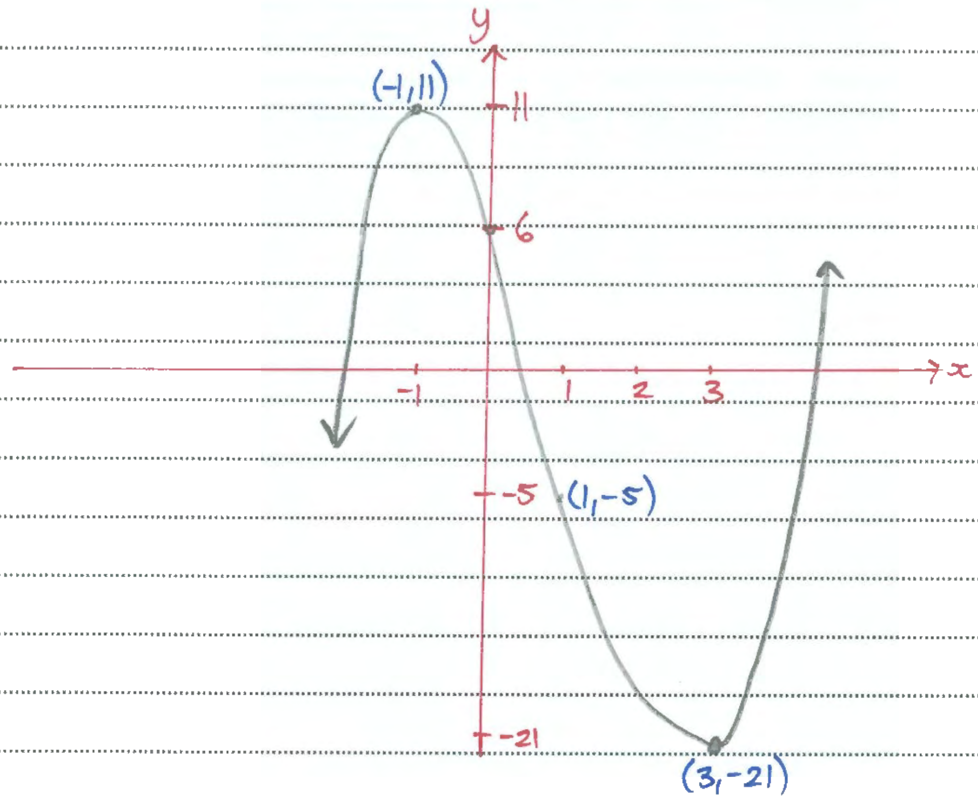
x	0	1	2
$f''(x)$	-6	0	6

$\therefore$ As concavity changes, $(1, -5)$ is a point of inflexion

Question 27 continues on page 17

Question 27 continued

(c) Sketch the graph of $y = f(x)$, labelling the stationary points and the y-intercept. Do not attempt to find the x-intercepts. 1



Question 28 (2 marks)

Differentiate $log_3 x^3$ 2

$= \frac{d}{dx}\left(\frac{\log_e x^3}{\log_e 3}\right)$

$= \frac{1}{\log_e 3} \cdot \frac{d}{dx}\left(\log_e x^3\right)$

$= \frac{1}{\log_e 3} \times \frac{3x^2}{x^3}$

$= \frac{3}{x \log_e 3}$

Question 29 (1 mark)

$\int \frac{x}{x^2-5}\, dx$ 1

$= \frac{1}{2} \ln(x^2-5) + C$

Question 30 (3 marks)

Determine the equation of a curve given by $\frac{d^2y}{dx^2} = 18x + 4$ if it is known that $(1, -2)$ is a stationary point on the curve. 3

$\frac{d^2y}{dx^2} = 18x + 4$

$\frac{dy}{dx} = 9x^2 + 4x + c$ $\quad \frac{dy}{dx} = 0$ when $x = 1$

$9(1)^2 + 4(1) + c = 0$

$\therefore c = -13$

$\frac{dy}{dx} = 9x^2 + 4x - 13$

$y = \frac{9x^3}{3} + \frac{4x^2}{2} - 13x + c$

$y = 3x^3 + 2x^2 - 13x + c$ $\quad$ sub $x = 1, y = -2$

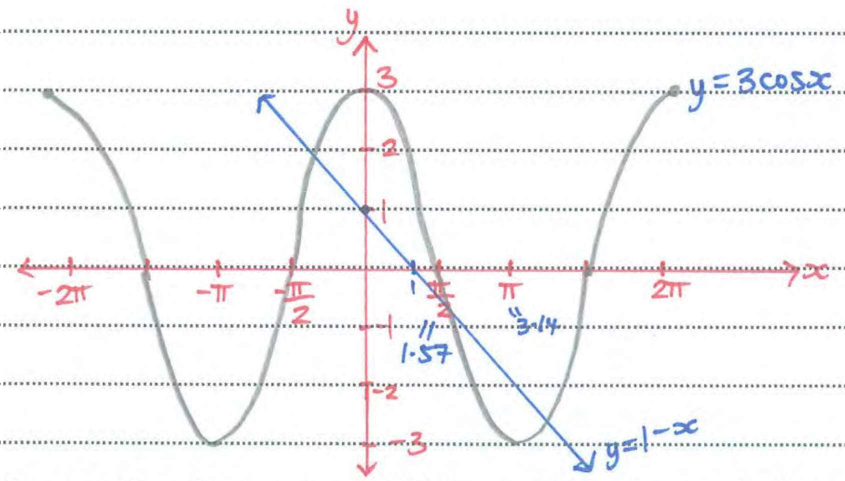
$-2 = 3 + 2 - 13 + c$

$\therefore c = 6$

$\therefore y = 3x^3 + 2x^2 - 13x + 6$

Question 31 (3 marks)

(a) Sketch $y = 3cosx$ in the domain $-2\pi \le x \le 2\pi$ 2



(b) Hence or otherwise, how many solutions are there to the equation $3cosx = 1 - x$? 1

Sketch $y = 1 - x$

y-intercept: $y = 1$

x-intercept: $x = 1$

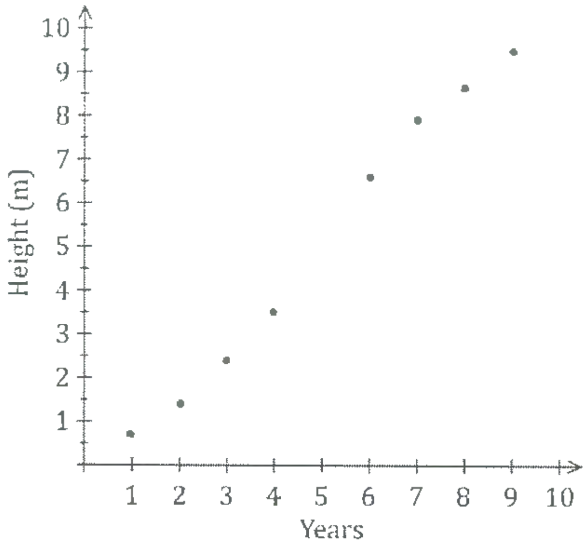
3 solutions (as there are 3 points of intersection)

Question 32 (4 marks)

Tim is a scientist studying the growth of a particular tree over several years. The data he recorded is shown in the table below.

Years since planting, t	1	2	3	4	6	7	8	9
Height of tree, H metres	0.7	1.4	2.4	3.5	6.6	7.9	8.7	9.5

A scatterplot of the data is shown below.



(a) What is Pearson's correlation coefficient? Answer correct to 4 decimal places. 1

$r = 0.9952$

(b) Find the equation of the least-squares line of best fit in terms of years (t) and height (h). Answer correct to 2 decimal places. 1

$y = BX + A$ $A = -0.85$

$B = 1.19$

$\therefore h = 1.19t - 0.85$

(c) Tim did not record the tree's height after five years. Predict the height after five years, correct to 1 decimal place. 1

when $t = 5$, $h = 1.19(5) - 0.85$

$= 5.1\text{m}$

(d) Estimate how many years it will take for the tree to reach a height of 24 metres. Answer correct to 1 decimal place. 1

when $h = 24$, $24 = 1.19t - 0.85$

$24.85 = 1.19t$

$t = 20.9$ years

Question 33 (2 marks)

An object is in motion along a line. The velocity, as measured at several instants of time, is given in the following table. Use the trapezoidal rule with 5 function values to approximate the distance travelled from $t = 0$ to $t = 4$ seconds. 2

t (s)	0	1	2	3	4
v (m/s)	3.2	2.7	2.9	4.0	4.7

$A_T = \frac{1}{2}[3\cdot2 + 4\cdot7 + 2(2\cdot7 + 2\cdot9 + 4\cdot0)]$

$= 13\cdot55$

Question 34 (3 marks)

A large tank of liquid which contains L litres of a toxic chemical is being drained. The amount of chemical in the tank over time t minutes, is given by:

$$L = 110(20 - t)^2$$

(a) At what rate is the chemical draining out of the tank after 5 minutes? 2

$L = 110(20-t)^2$

$\frac{dL}{dt} = 220(20-t)^1 \times -1$

$= -220(20-t)$

when $t = 5$, $\frac{dL}{dt} = -220(20-5)$

$= -3300$ L/min

(b) How long will it take for the tank to be completely empty? 1

$L = 0$

$110(20-t)^2 = 0$

$(20-t)^2 = 0$

$20 - t = 0$

$t = 20$ mins

Question 35 (4 marks)

A factory is producing laptops. The annual production, M laptops at time t years, is given by:

$M = M_0 e^{kt}$

Initially the production at the factory was 2000 laptops per annum.

Five years later it had increased to 3200 laptops per annum.

(a) Find the values of M_0 and k (Answer correct to three decimal places). 2

$t = 0, M = 2000$

$2000 = M_0 e^0$

$\therefore M_0 = 2000$

$M = 2000e^{kt}$ when $t = 5, M = 3200$

$3200 = 2000e^{5k}$

$1.6 = e^{5k}$

$\ln 1.6 = \ln e^{5k}$

$\ln 1.6 = 5k$

$k = \frac{\ln 1.6}{5}$

$= 0.094$ (3d.p.)

(b) How many years will it take for the production to double its original output? Answer correct to one decimal place. 2

$M = 4000$

$4000 = 2000e^{\frac{\ln 1.6}{5}t}$

$2 = e^{\frac{\ln 1.6}{5}t}$

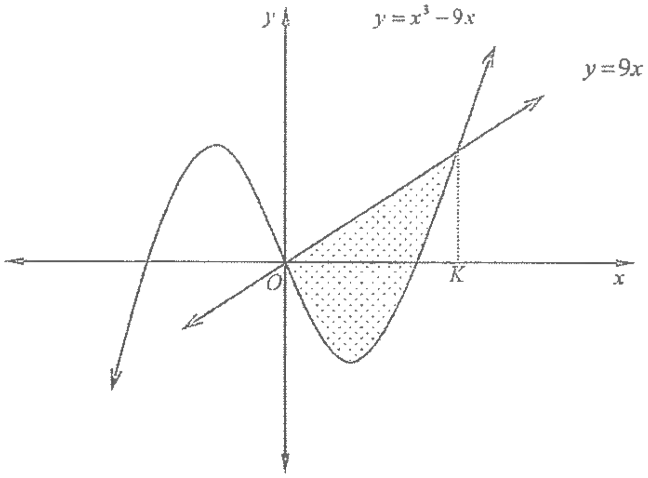
$\ln 2 = \ln e^{\frac{\ln 1.6}{5}t}$

$\ln 2 = \frac{\ln 1.6}{5}t$

$t = 7.37$

$\therefore t = 7.4$ years

Question 36 (5 marks)



The graphs of the functions $y = 9x$ and $y = x^3 - 9x$ are shown above.

The graphs intersect at $x = 0$ and $x = K$ for $x \geq 0$.

(a) By solving the two equations simultaneously, show that $K = 3\sqrt{2}$ **2**

$x^3 - 9x = 9x$

$x^3 - 18x = 0$

$x(x^2 - 18) = 0$

$x = 0$ or $x^2 - 18 = 0$

$x^2 = 18$

$x = \pm\sqrt{18}$

$= \pm 3\sqrt{2}$ but $K > 0$

$\therefore K = 3\sqrt{2}$

(b) Hence find the shaded area in the diagram above. **3**

$A = \int_0^{3\sqrt{2}} 9x - (x^3 - 9x)\,dx$

$= \int_0^{3\sqrt{2}} 9x - x^3 + 9x\,dx$

$= \int_0^{3\sqrt{2}} 18x - x^3\,dx$

$= \left[\frac{18x^2}{2} - \frac{x^4}{4}\right]_0^{3\sqrt{2}}$

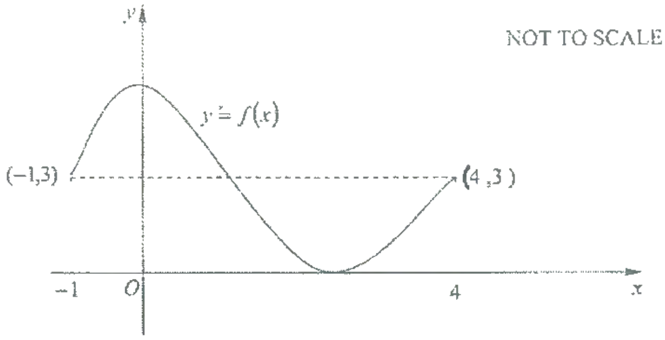
$= \left[9(3\sqrt{2})^2 - \frac{(3\sqrt{2})^4}{4}\right] - [0]$

$= 162 - 81$

$= 81$ units^2

Question 37 (2 marks)

The graph below represents the function $y = f(x)$.



If $\int_{-1}^{4} f(x)\,dx = \frac{15}{2}$, find the value of $\int_{-1}^{4} [f(x) + 4]\,dx$ 2

$= \int_{-1}^{4} f(x)\,dx + \int_{-1}^{4} 4\,dx$

$= \frac{15}{2} + [4x]_{-1}^{4}$

$= \frac{15}{2} + [16 - 4(-1)]$

$= \frac{15}{2} + 20$

$= 27.5$

Question 38 (4 marks)

(a) Show that $\frac{1}{2x-5} - \frac{1}{2x+5} = \frac{10}{4x^2-25}$ 1

$LHS = \frac{1}{2x-5} - \frac{1}{2x+5}$

$= \frac{2x+5-(2x-5)}{(2x-5)(2x+5)}$

$= \frac{2x+5-2x+5}{4x^2-25}$

$= \frac{10}{4x^2-25}$

$= RHS$

Question 38 continues on page 24

Question 38 continued

(b) Hence find $\int \frac{dx}{4x^2-25}$ Leave your answer in simplest form. 3

$\frac{1}{10}\int \frac{dx}{4x^2-25} \times 10 = \frac{1}{10}\int \frac{1}{2x-5} - \frac{1}{2x+5}\,dx$

$= \frac{1}{10}\left[\frac{1}{2}\ln(2x-5) - \frac{1}{2}\ln(2x+5)\right] + C$

$= \frac{1}{20}\left[\ln(2x-5) - \ln(2x+5)\right] + C$

$= \frac{1}{20}\ln\left(\frac{2x-5}{2x+5}\right) + C$

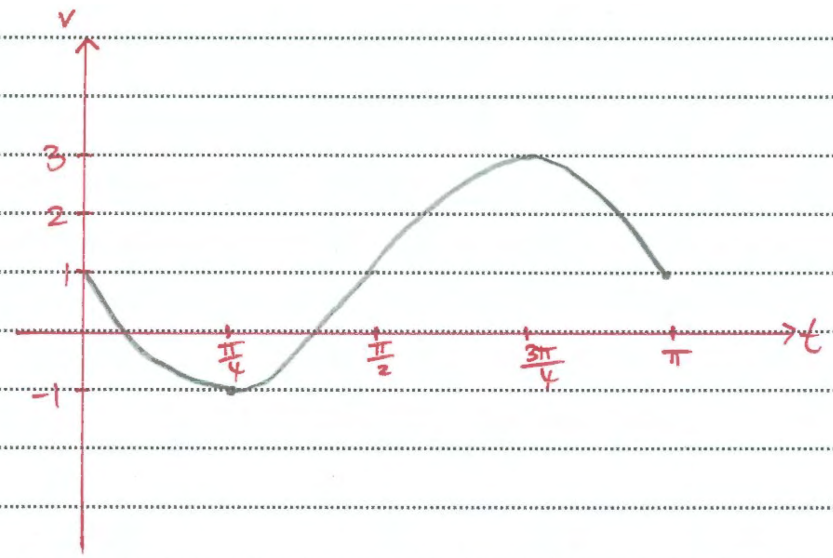
Question 39 (8 marks)

A particle is moving in a straight line and its velocity is given by:

$$v = 1 - 2sin2t \text{ for } 0 \leq t \leq \pi$$

where v is measured in metres per second and t in seconds.

(a) Sketch the graph of v as a function of t for $0 \leq t \leq \pi$ 2



(b) At what time(s) is particle's acceleration zero? 1

$t = \frac{\pi}{4}, \frac{3\pi}{4}$

Question 39 continues on page 25

Question 39 continued

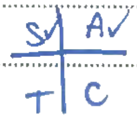
(c) When is the particle at rest? 2

At rest when $v=0$

$1-2\sin 2t=0$

$2\sin 2t=1$

$\sin 2t=\frac{1}{2}$



$2t=\frac{\pi}{6},\frac{5\pi}{6}$

$t=\frac{\pi}{12},\frac{5\pi}{12}$

(d) Initially the particle is at the origin.
Find the displacement function x as a function of t 2

$t=0, x=0$ $\quad x=\int 1-2\sin 2t\ dt$

$x=t+\cos 2t+c$

$0=0+\cos 0+c$

$0=1+c$

$c=-1$

$\therefore x=t+\cos 2t-1$

(e) What is the position of the particle when $t=\frac{\pi}{3}$? Leave your answer in exact form. 1

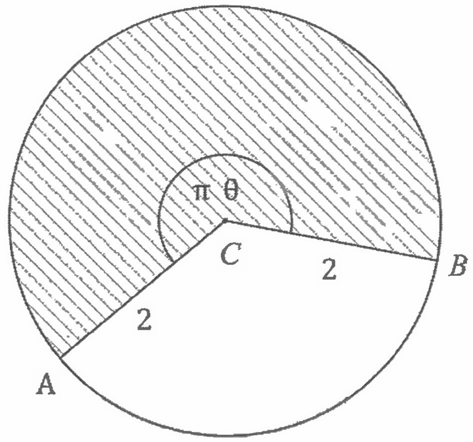
when $t=\frac{\pi}{3}$, $x=\frac{\pi}{3}+\cos\left(2\times\frac{\pi}{3}\right)-1$

$=\frac{\pi}{3}+\cos\left(\frac{2\pi}{3}\right)-1$

$=\frac{\pi}{3}-\frac{1}{2}-1$

$=\frac{\pi}{3}-\frac{3}{2}$

Question 40 (7 marks)



The angle at the centre C of a circle of radius 2 cm is $\pi\theta$ radians, $0 < \theta < 2$, as shown on the diagram above.

(a) Write down the length of the arc of the shaded sector 1

$l = r\theta$

$= 2\pi\theta$

(b) The sector is cut from the circle along the radii CA and CB and folded to make a cone. Find the radius of the cone. 1

circumference $= 2\pi\theta$

$C = 2\pi r$

$\therefore 2\pi r = 2\pi\theta$

$r = \theta$

(c) Show that the volume of the cone is given by $V = \frac{\pi}{3}\sqrt{4\theta^4 - \theta^6}$ 1

(The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2\sqrt{l^2 - r^2}$ where l is the slant height)

$V = \frac{1}{3}\pi r^2\sqrt{l^2 - r^2}$ $\quad l = 2$

$= \frac{1}{3}\pi\,\theta^2\sqrt{2^2 - \theta^2}$

$= \frac{\pi}{3}\theta^2\sqrt{4 - \theta^2}$

$= \frac{\pi}{3}\sqrt{\theta^4(4 - \theta^2)}$

$= \frac{\pi}{3}\sqrt{4\theta^4 - \theta^6}$ as required

Question 40 continues on page 27

Question 40 continued

(d) Find the value of θ, to 2 decimal places, for which the volume of the cone is maximised. **4**

$V = \frac{\pi}{3}(4\theta^4 - \theta^6)^{\frac{1}{2}}$

Maximum when $V' = 0$

$V' = \frac{\pi}{6}(4\theta^4 - \theta^6)^{-\frac{1}{2}} \cdot (16\theta^3 - 6\theta^5)$

$0 = \frac{\pi(16\theta^3 - 6\theta^5)}{6\sqrt{4\theta^4 - \theta^6}}$

$0 = \pi(16\theta^3 - 6\theta^5)$

$16\theta^3 - 6\theta^5 = 0$

$2\theta^3(8 - 3\theta^2) = 0$

$\theta = 0$ or $8 - 3\theta^2 = 0$

$\theta \neq 0$ $\quad 3\theta^2 = 8$

$\theta^2 = \frac{8}{3}$ $\quad \theta > 0$

$\theta = \sqrt{\frac{8}{3}}$

$\theta = 1.63$ (2d.p.)

Check nature:

θ	1	1.63	1.8
V'	3.02	0	-3.72
	/	—	\

$\therefore$ Maximised when $\theta = 1.63$

End of paper